

Reminder on the Wigner-Eckart theorem

In our angular momentum lectures on the Wigner-Eckart theorem we developed two equations. One for the general expectation value that depends also on the magnetic quantum numbers

$$\langle \Phi_M^J | T_\mu^\lambda | \Phi_{M'}^{J'}
angle \equiv (-1)^{J-M} \begin{pmatrix} J & \lambda & J' \\ -M & \mu & M' \end{pmatrix} \langle \Phi^J || T^\lambda || \Phi^{J'}
angle,$$

and one for the reduced matrix elements in terms of

$$\langle \Phi^J || \mathcal{T}^{\lambda} || \Phi^{J'}
angle \equiv \sum_{M,\mu,M'} (-1)^{J-M} \left(egin{array}{cc} J & \lambda & J' \ -M & \mu & M' \end{array}
ight) \langle \Phi^J_M | \mathcal{T}^{\lambda}_{\mu} | \Phi^{J'}_{M'}
angle$$

Why the Wigner-Eckart theorem?

Unless we have observables which depend on the magnetic quantum numbers, the degeneracy given by these quantum numbers is not seen experimentally. The typical situation when we perform shell-model calculations is that the results depend on the magnetic quantum numbers. The reason for this is that it is easy to implement the Pauli principle for many particles when we work in what we dubbed for *m*-scheme.

A resulting state in a shell-model calculations will thus depend on the total value of M defined as

$$M = \sum_{i=1}^{A} m_{j_i}.$$

A shell model many-body state is given by a linear combination of Slater determinants $|\Phi_i\rangle$. That is, for some conserved quantum numbers λ we have

 $|Psi_{\lambda}\rangle = \sum C_i |\Phi_i\rangle,$

Why the Wigner-Eckart theorem, representing a shell-model state In second quantization, our ansatz for a state like the ground state is $|\Phi_0 angle = \left(\prod_{i\leq F} \hat{a}^{\dagger}_i ight)|0 angle,$ where the index i defines different single-particle states up to the Fermi level. We have assumed that we have N fermions. A given one-particle-one-hole (1p1h) state can be written as $|\Phi_i^a\rangle = \hat{a}_a^\dagger \hat{a}_i |\Phi_0\rangle,$ while a 2p2h state can be written as

$$|\Phi^{ab}_{ij}
angle=\hat{a}^{\dagger}_{a}\hat{a}^{\dagger}_{b}\hat{a}_{j}\hat{a}_{i}|\Phi_{0}
angle,$$

and a general NpNh state as

 $|\Phi_{iik}^{abc...}\rangle = \hat{a}_{a}^{\dagger}\hat{a}_{b}^{\dagger}\hat{a}_{c}^{\dagger}\dots\hat{a}_{k}\hat{a}_{i}\hat{a}_{i}|\Phi_{0}\rangle$

Representing a shell-model state
A general shell-model many-body state

$$|Psi_{\gamma}\rangle = \sum_{i} C_{i}|\Phi_{i}\rangle,$$

can be expanded as
 $|\Psi_{\gamma}\rangle = C_{0}|\Phi_{0}\rangle + \sum_{ai} C_{i}^{a}|\Phi_{i}^{a}\rangle + \sum_{abij} C_{ij}^{ab}|\Phi_{ij}^{ab}\rangle + \dots$

Representing a shell-model state and one-body operators
A one-body operator represented by a spherical tensor of rank
$$\lambda$$
 is
given as
 $O^{\lambda}_{\mu} = \sum_{pq} \langle p | O^{\lambda}_{\mu} | q \rangle a^{\dagger}_{p} a_{q},$
meaning that when we compute a transition amplitude
 $\langle \Psi_{\delta} | O^{\lambda}_{\mu} | \Psi_{\gamma} \rangle = \sum_{ij} C^{*}_{\delta i} C_{\gamma j} \langle \Phi_{i} | O^{\lambda}_{\mu} | \Phi_{j} \rangle,$
we need to compute
 $\langle \Phi_{i} | O^{\lambda}_{\mu} | \Phi_{j} \rangle.$

Rewriting the transition amplitude

We want to rewrite

in terms of the reduced matrix element only. Let us introduce the relevant quantum numbers for the states Φ_i and Φ_j . We include only the relevant ones. We have then in *m*-scheme

 $\langle \Phi_i | O^{\lambda}_{\mu} | \Phi_j \rangle,$

$$\langle \Phi_i | O^{\lambda}_{\mu} | \Phi_j \rangle = \sum_{pq} \langle p | O^{\lambda}_{\mu} | q \rangle \langle \Phi^J_M | a^{\dagger}_p a_q | \Phi^{J'}_{M'} \rangle.$$

With a shell-model *m*-scheme basis it is straightforward to compute these amplitudes. However, as mentioned above, if we wish to related these elements to experiment, we need to use the Wigner-Eckart theorem and express the amplitudes in terms of reduced matrix elements.



We can rewrite the above transition amplitude using the Wigner-Eckart theorem. Our first step is to rewrite the one-body operator in terms of reduced matrix elements. Since the operator is a spherical tensor we need that the annihilation operator is rewritten as (where q represents j_q , m_q etc)

$$\tilde{a}_q = (-1)^{j_q - m_q} a_{j_q, m_q}$$

 $O^{\lambda}_{\mu} = \sum_{pq} \langle p || O^{\lambda} || q
angle (-1)^{j_{
ho} - m_{
ho}} \left(egin{array}{cc} j_{
ho} & \lambda & j_{q} \ -m_{
ho} & \mu & m_{q} \end{array}
ight) a^{\dagger}_{
ho} a_{q}.$

The operator
$$O^\lambda_\mu = \sum_{pq} \langle p|O^\lambda_\mu|q\rangle a^\dagger_p a_q,$$

is rewritten using the Wigner-Eckart theorem as

Rewriting the transition amplitude, second step
We have

$$\mathcal{O}_{\mu}^{\lambda} = \sum_{pq} \langle p || \mathcal{O}^{\lambda} || q \rangle (-1)^{j_p - m_p} \begin{pmatrix} j_p & \lambda & j_q \\ -m_p & \mu & m_q \end{pmatrix} a_p^{\dagger} a_q.$$
We then single out the sum over m_p and m_q only and define the recoupled one-body part of the operator as

$$\lambda^{-1} \left[a_{j_p}^{\dagger} \tilde{g}_{i_q} \right]_{\mu}^{\lambda} = \sum_{m_p, m_q} (-1)^{j_p - m_p} \begin{pmatrix} j_p & \lambda & j_q \\ -m_p & \mu & m_q \end{pmatrix} a_p^{\dagger} a_q,$$
with $\lambda = \sqrt{2\lambda + 1}$. This gives the following expression for the one-body operator

$$\mathcal{O}_{\mu}^{\lambda} = \sum_{j_p j_q} \langle p || \mathcal{O}^{\lambda} || q \rangle \lambda^{-1} \left[a_{j_p}^{\dagger} \tilde{g}_{j_q} \right]_{\mu}^{\lambda}.$$

Rewriting the transition amplitude, third step
With
$$\begin{array}{l}
\mathcal{O}_{\mu}^{\lambda} = \sum_{j_{p\bar{l}q}} \langle p||\mathcal{O}^{\lambda}||q\rangle\lambda^{-1} \left[a_{j_{p}}^{\dagger}\tilde{a}_{j_{q}}\right]_{\mu}^{\lambda},\\
\text{we can write}
\\
\langle \Phi_{M}^{J}|\mathcal{O}_{\mu}^{\lambda}|\Phi_{M'}^{J'}\rangle = \sum_{pq} \langle p|\mathcal{O}_{\mu}^{\lambda}|q\rangle\langle \Phi_{M}^{J}|a_{p}^{\dagger}a_{q}|\Phi_{M'}^{J'}\rangle,\\
\text{as}
\\
\langle \Phi_{M}^{J}|\mathcal{O}_{\mu}^{\lambda}|\Phi_{M'}^{J'}\rangle = \sum_{j_{p\bar{l}q}} \langle p||\mathcal{O}^{\lambda}||q\rangle\langle \Phi_{M}^{J}|\lambda^{-1} \left[a_{j_{p}}^{\dagger}\tilde{a}_{j_{q}}\right]_{\mu}^{\lambda}|\Phi_{M'}^{J'}\rangle.\\
\text{We have suppressed the summation over quantum numbers like}\\
n_{p}, n_{q} \text{ etc.}
\end{array}$$

Rewriting the transition amplitude, final step
Using the Wigner-Eckart theorem

$$\langle \Phi_{M}^{J} | \Theta_{\mu}^{\lambda} | \Phi_{M'}^{J'} \rangle \equiv (-1)^{J-M} \begin{pmatrix} J & \lambda & J' \\ -M & \mu & M' \end{pmatrix} \langle \Phi^{J} | | O^{\lambda} | | \Phi^{J'} \rangle$$
,
we can then define
 $\langle \Phi^{J} | | O^{\lambda} | | \Phi^{J'} \rangle = \lambda^{-1} \sum_{j_{R} j_{q}} \langle p | | O^{\lambda} | | q \rangle \langle \Phi_{M}^{J} | | \left[\tilde{a}_{j_{p}}^{\dagger} \tilde{a}_{j_{q}} \right]^{\lambda} | | \Phi_{M'}^{J'} \rangle$.
The quantity to the left in the last equation is normally called the
transition amplitude or in case of a decay process, simply the decay
amplitude. The quantity $\langle \Phi_{M}^{J} | \lambda^{-1} \left[a_{j_{p}}^{\dagger} \tilde{a}_{j_{q}} \right]^{\lambda} | \Phi_{M'}^{J'} \rangle$ is called the
one-body transition density while the corresponding reduced one is
simply called the reduced one-body transition density. The
transition densities characterize the many-nucleon properties of the
initial and final states. They do not carry information about the
transition operator beyond its one-body character. Finally, note

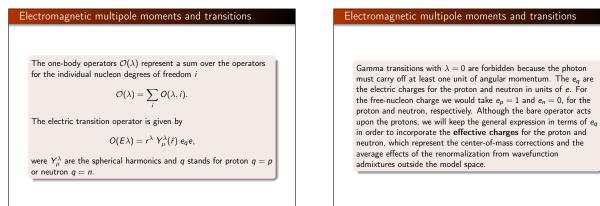
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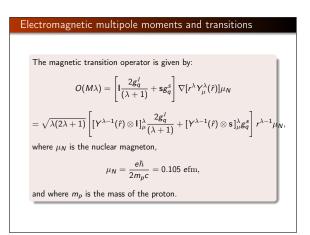
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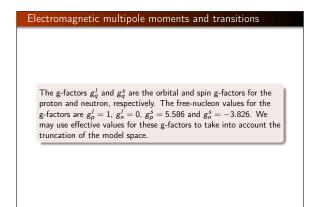
Electromagnetic multipole moments and transitions
The reduced transition probability *B* is defined in terms of reduced
matrix elements of a one-body operator by

$$B(i \rightarrow f) = \frac{\langle J_f || \mathcal{O}(\lambda) || J_i \rangle^2}{(2J_i + 1)}.$$
With our definition of the reduced matrix element,

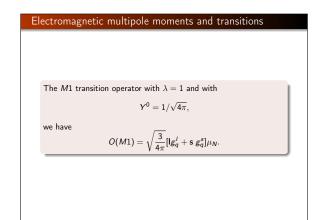
$$\langle J_f || \mathcal{O}(\lambda) || J_i \rangle^2 = \langle J_i || \mathcal{O}(\lambda) || J_f \rangle^2,$$
the transition probability *B* depends upon the direction of the
transition by the factor of $(2J_i + 1)$. For electromagnetic transitions
 J_i is that for the higher-energy initial state. But in Coulomb
excitation the initial state is usually taken as the ground state, and
it is normal to use the notation $B(\uparrow)$ for transitions from the
ground state.





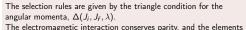


Electromagnetic multipole moments and transitions
The most common types of transitions are E1, E2 and M1. The E1 transition operator is given by $\lambda = 1$
$O(E1)=rY^{(1)}_\mu(\hat{r})e_qe=\sqrt{rac{3}{4\pi}}re_qe.$
The <i>E</i> 2 transition operator with $\lambda = 2$
$O(E2) = r^2 Y^{(2)}_{\mu}(\hat{r}) e_q e,$





Electromagnetic multipole moments and transitions



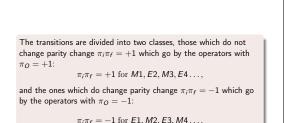
of the operators for $E\lambda$ and $M\lambda$ can be classified according to their transformation under parity change

 $\hat{P}\hat{O}\hat{P}^{-1} = \pi_O\hat{O},$

where we have $\pi_O = (-1)^{\lambda}$ for Y^{λ} , $\pi_O = -1$ for the vectors \mathbf{r} , ∇ and \mathbf{p} , and $\pi_O = +1$ for the pseudo vectors $\mathbf{l} = \mathbf{r} \times \mathbf{p}$ and σ . For a given matrix element we have:

 $\langle \Psi_f | \mathcal{O} | \Psi_i \rangle = \langle \Psi_f | P^{-1} P \mathcal{O} P^{-1} P | \Psi_i \rangle = \pi_i \pi_f \pi_O \langle \Psi_f | \mathcal{O} | \Psi_i \rangle.$

The matrix element will vanish unless $\pi_i \pi_f \pi_O = +1$.



Electromagnetic multipole moments and transitions The electromagnetic moment operator can be expressed in terms of the electromagnetic transition operators. By the parity selection rule of the moments are nonzero only for *M*1, *E*2, *M*3, *E*4, The most common are: $\mu = \sqrt{\frac{4\pi}{3}} \langle J, M = J | \mathcal{O}(M1) | J, M = J \rangle = \sqrt{\frac{4\pi}{3}} \left\{ \begin{array}{cc} J & 1 & J \\ -J & 0 & J \end{array} \right\} \langle J || \mathcal{O}(M1) | J, M = J \rangle = \sqrt{\frac{4\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ -J & 0 & J \end{array} \right) \langle J || \mathcal{O}(M2) | J, M = J \rangle = \sqrt{\frac{16\pi}{5}} \left(\begin{array}{cc} J & 2 & J \\ J &$

Electromagnetic multipole moments and transitions

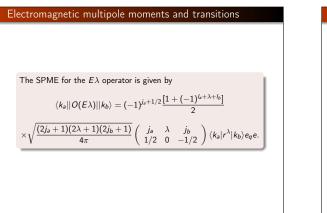
Electromagnetic transitions and moments depend upon the reduced nuclear matrix elements $\langle f || O(\lambda) || i \rangle$. These can be expressed as a sum over one-body transition densities (OBTD) times single-particle matrix elements

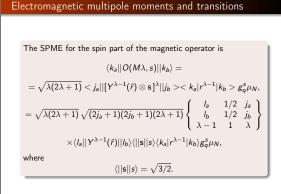
$$\langle f || \mathcal{O}(\lambda) || i \rangle = \sum_{k_{\alpha}k_{\beta}} \text{OBTD}(fik_{\alpha}k_{\beta}\lambda) \langle k_{\alpha} || \mathcal{O}(\lambda) || k_{\beta} \rangle$$

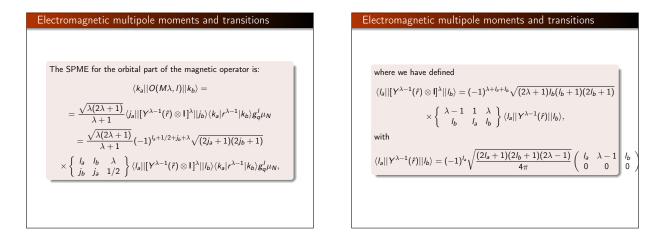
where the OBTD is given by

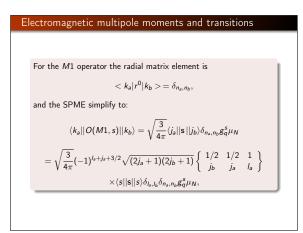
$$OBTD(fik_{\alpha}k_{\beta}\lambda) = \frac{\langle f || [a_{k_{\alpha}}^{+} \otimes \tilde{a}_{k_{\beta}}]^{\lambda} || i}{\sqrt{(2\lambda+1)}}$$

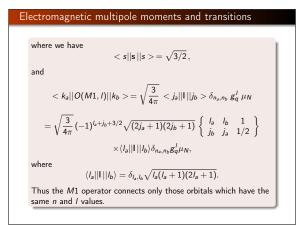
The labels *i* and *f* are a short-hand notation for the initial and final state quantum numbers $(n\omega_i J_i)$ and $(n\omega_f J_f)$, respectively. Thus the problem is divided into two parts, one involving the nuclear structure dependent one-body transition densities OBTD, and the other involving the reduced single-particle matrix elements (SPME).

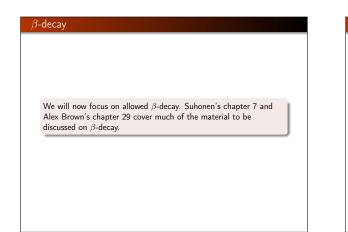












β -decay

The allowed beta decay rate W between a specific set of initial and final states is given by

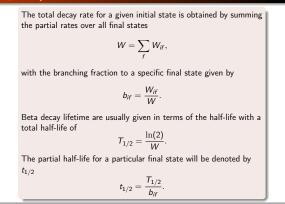
$$W_{i,f} = (f/K_o) \left[g_V^2 B_{i,f}(F_{\pm}) + g_A^2 B_{i,f}(GT_{\pm}) \right],$$

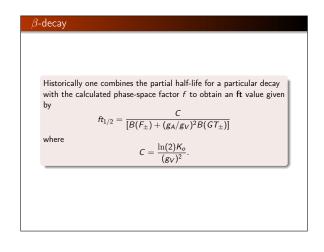
where f is dimensionless three-body phase-space factor which depends upon the beta-decay Q value, and K_o is a specific combination of fundamental constants

$$K_o = \frac{2\pi^3\hbar^7}{m_e^5c^4} = 1.8844 \times 10^{-94} \mathrm{erg}^2 \mathrm{cm}^6 \mathrm{s}.$$

The \pm signrefer to β_{\pm} decay of nucleus (A_i, Z_i) into nucleus ($A_i, Z_i \mp 1$). The weak-interaction vector (V) and axial-vector (A) coupling constants for the decay of neutron into a proton are denoted by g_V and g_A , respectively.

β -decay





eta-decay

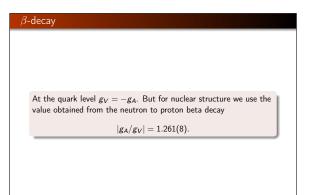
One often compiles the allowed beta decay in terms of a **logft** which stands for \log_{10} of the $f_{1/2}$ value. The values of the coupling constants for Fermi decay, g_V , and Gamow-Teller decay, g_A are obtained as follows. For a $0^+ \rightarrow 0^+$ nuclear transition B(GT) = 0, and for a transition between T = 1 analogue states with B(F) = 2 we find

 $C = 2t_{1/2}f.$

The partial half-lives and Q values for several $0^+ \to 0^+$ analogue transitions have been measured to an accuracy of about one part in 10000. With phase space factors one obtains

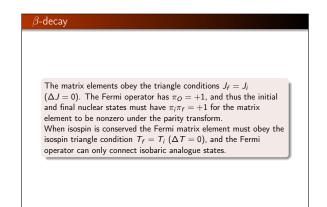
C = 6170(4)

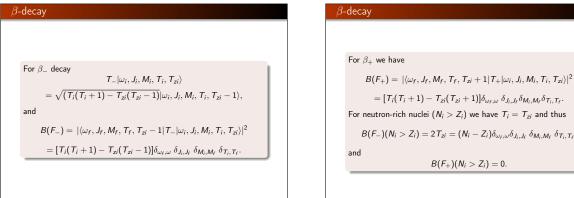
This result, together with the value of K_o can be used to obtain g_V .

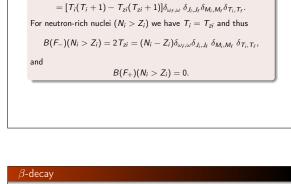


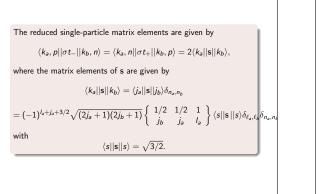
β -decay

The operator for Fermi beta decay in terms of sums over the nucleons is
$\mathcal{O}(F_{\pm}) = \sum_{k} t_{k\pm}.$
The matrix element is
$B(F) = \langle f T_{\pm} i \rangle ^2,$
where $T_{\pm} = \sum_k t_{\pm}$
is the total isospin raising and lowering operator for total isospin constructed out of the basic nucleon isospin raising and lowering
operators
$t_{-} n angle = ho angle \qquad t_{-} ho angle = 0,$
and
$t_+ p angle= n angle, \ \ t_+ n angle=0.$









 β -decay

The matrix elements of **s** has the selection rules δ_{ℓ_a,ℓ_b} and δ_{n_a,n_b} . Thus the orbits which are connected by the GT operator are very selective; they are those in the same major oscillator shell with the same ℓ value. The matrix elements such as $1s_{1/2} - 0d_{3/2}$ which have the allowed Δj coupling but are zero due to the $\Delta \ell$ coupling are called *l*-forbidden matrix elements.

β -decay Sum rules for Fermi and Gamow-Teller matrix elements can be obtained easily. The sum rule for Fermi is obtained from the sum $\sum_{f} \left[B_{fi}(F_{-}) - B_{fi}(F_{+}) \right] = \sum_{f} \left[\left| \langle f | T_{-} | i \rangle \right|^{2} - \left| \langle f | T_{+} | i \rangle \right|^{2} \right]$ The final states f in the T_- matrix element go with the $Z_f = Z_i + 1$ nucleus and those in the T_+ matrix element to with the $Z_f = Z_i - 1$ nucleus. One can explicitly sum over the final states to obtain $\sum \left[\langle i | T_{+} | f \rangle \langle f | T_{-} | i \rangle - \langle i | T_{-} | f \rangle \langle f | T_{+} | i \rangle \right]$ $= \langle i|T_+T_- - T_-T_+|i\rangle = \langle i|2T_z|i\rangle = (N_i - Z_i).$

β -decay

The sum rule for Gamow-Teller is obtained as follows

$$\begin{split} \sum_{f,\mu} |\langle f| \sum_{k} \sigma_{k,\mu} t_{k-} |i\rangle|^2 &- \sum_{f,\mu} |\langle f| \sum_{k} \sigma_{k,\mu} t_{k+} |i\rangle|^2 \\ &= \sum_{f,\mu} \langle i| \sum_{k} \sigma_{k,\mu} t_{k+} |f\rangle \langle f| \sum_{k'} \sigma_{k',\mu} t_{k'-} |i\rangle \\ &- \sum_{f,\mu} \langle i| \sum_{k} \sigma_{k,\mu} t_{k-} |f\rangle \langle f| \sum_{k'} \sigma_{k',\mu} t_{k'+} |i\rangle \\ &= \sum_{\mu} \left[\langle i| \left(\sum_{k} \sigma_{k,\mu} t_{k+} \right) \left(\sum_{k'} \sigma_{k',\mu} t_{k'-} \right) - \left(\sum_{k} \sigma_{k,\mu} t_{k-} \right) \left(\sum_{k'} \sigma_{k',\mu} t_{k'} t_{k'-} \right) \right] \\ &= \sum_{\mu} \langle i| \sum_{k} \sigma_{k,\mu}^2 [t_{k+} t_{k-} - t_{k-} t_{k+}] |i\rangle \\ &= 3 \langle i| T_+ T_- - T_- T_+ |i\rangle = 3 \langle i| 2T_z |i\rangle = 3(N_i - Z_i). \end{split}$$

decay

$$\beta$$
-decay
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Core-polarization

We need to say something about so-called core-polarization effects. To do this, we have to introduce elements from many-body perturbation theory.

We assume here that we are only interested in the ground state of the system and expand the exact wave function in term of a series of Slater determinants

$$|\Psi_0
angle = |\Phi_0
angle + \sum_{m=1}^\infty C_m |\Phi_m
angle$$

where we have assumed that the true ground state is dominated by the solution of the unperturbed problem, that is

$$\hat{H}_0 |\Phi_0\rangle = W_0 |\Phi_0\rangle.$$

The state $|\Psi_0\rangle$ is not normalized, rather we have used an intermediate normalization $\langle\Phi_0|\Psi_0\rangle=1$ since we have $\langle\Phi_0|\Phi_0\rangle=1.$

Electromagnetic multipole moments and transitions

The Schroedinger equation is

$$\hat{H}|\Psi_0\rangle = E|\Psi_0\rangle,$$

and multiplying the latter from the left with $\langle \Phi_0 |$ gives

 $\langle \Phi_0 | \hat{H} | \Psi_0 \rangle = E \langle \Phi_0 | \Psi_0 \rangle = E,$

and subtracting from this equation

 $\langle \Psi_0 | \hat{H}_0 | \Phi_0 \rangle = W_0 \langle \Psi_0 | \Phi_0 \rangle = W_0,$

and using the fact that the both operators \hat{H} and \hat{H}_0 are hermitian results in

 $\Delta E = E - W_0 = \langle \Phi_0 | \hat{H}_I | \Psi_0 \rangle,$

which is an exact result. We call this quantity the correlation energy.

Electromagnetic multipole moments and transitions

 $|\Psi_0
angle = (\hat{P} + \hat{Q})|\Psi_0
angle = |\Phi_0
angle + \hat{Q}|\Psi_0
angle.$

 $\left(\omega - \hat{H}_{0}
ight) |\Psi_{0}
angle = \left(\omega - E + \hat{H}_{I}
ight) |\Psi_{0}
angle,$

Going back to the Schrödinger equation, we can rewrite it as,

We can thus rewrite the exact wave function as

adding and a subtracting a term $\omega |\Psi_0\rangle$ as

where ω is an energy variable to be specified later.

Electromagnetic multipole moments and transitions

This equation forms the starting point for all perturbative derivations. However, as it stands it represents nothing but a mere formal rewriting of Schroedinger's equation and is not of much practical use. The exact wave function $|\Psi_0\rangle$ is unknown. In order to obtain a perturbative expansion, we need to expand the exact wave function in terms of the interaction \hat{H}_I . Here we have assumed that our model space defined by the operator \hat{P} is one-dimensional, meaning that

 $\hat{P} = |\Phi_0\rangle\langle\Phi_0|,$

 $\hat{Q} = \sum_{m=1}^{\infty} |\Phi_m\rangle \langle \Phi_m|.$

Electromagnetic multipole moments and transitions

We assume also that the resolvent of $\left(\omega-\dot{H}_0\right)$ exits, that is it has an inverse which defined the unperturbed Green's function as

$$\left(\omega-\hat{H}_0
ight)^{-1}=rac{1}{\left(\omega-\hat{H}_0
ight)}.$$

We can rewrite Schroedinger's equation as

$$|\Psi_0
angle = rac{1}{\omega - \hat{H}_0} \left(\omega - E + \hat{H}_I
ight) |\Psi_0
angle,$$

and multiplying from the left with \hat{Q} results in

$$\hat{Q}|\Psi_0
angle = rac{\hat{Q}}{\omega-\hat{H}_0}\left(\omega-E+\hat{H}_I
ight)|\Psi_0
angle,$$

which is possible since we have defined the operator \hat{Q} in terms of the eigenfunctions of $\hat{H}.$

Electromagnetic multipole moments and transitions

These operators commute meaning that

$$\hat{\mathcal{Q}} rac{1}{\left(\omega - \hat{\mathcal{H}}_0
ight)} \hat{\mathcal{Q}} = \hat{\mathcal{Q}} rac{1}{\left(\omega - \hat{\mathcal{H}}_0
ight)} = rac{\hat{\mathcal{Q}}}{\left(\omega - \hat{\mathcal{H}}_0
ight)}.$$

With these definitions we can in turn define the wave function as

$$|\Psi_0
angle = |\Phi_0
angle + rac{\hat{Q}}{\omega - \hat{H}_0} \left(\omega - E + \hat{H}_I
ight) |\Psi_0
angle$$

This equation is again nothing but a formal rewrite of Schrödinger's equation and does not represent a practical calculational scheme. It is a non-linear equation in two unknown quantities, the energy E and the exact wave function $|\Psi_0\rangle$. We can however start with a guess for $|\Psi_0\rangle$ on the right hand side of the last equation.

Electromagnetic multipole moments and transitions

The most common choice is to start with the function which is expected to exhibit the largest overlap with the wave function we are searching after, namely $|\Phi_0\rangle$. This can again be inserted in the solution for $|\Psi_0\rangle$ in an iterative fashion and if we continue along these lines we end up with

$$|\Psi_0\rangle = \sum_{i=0}^{\infty} \left\{ \frac{\hat{Q}}{\omega - \hat{H}_0} \left(\omega - E + \hat{H}_I \right) \right\}^i |\Phi_0\rangle,$$

for the wave function and

$$\Delta E = \sum_{i=0}^{\infty} \langle \Phi_0 | \hat{H}_I \left\{ \frac{\hat{Q}}{\omega - \hat{H}_0} \left(\omega - E + \hat{H}_I \right) \right\}^i | \Phi_0 \rangle,$$

which is now a perturbative expansion of the exact energy in terms of the interaction \hat{H}_l and the unperturbed wave function $|\Psi_0\rangle$.

Electromagnetic multipole moments and transitions

In our equations for $|\Psi_0\rangle$ and ΔE in terms of the unperturbed solutions $|\Phi_i\rangle$ we have still an undetermined parameter ω and a dependecy on the exact energy E. Not much has been gained thus from a practical computational point of view. In Brilluoin-Wigner perturbation theory it is customary to set $\omega=E$. This results in the following perturbative expansion for the energy ΔE

$$\Delta E = \sum_{i=0}^{\infty} \langle \Phi_0 | \hat{H}_I \left\{ \frac{\hat{Q}}{\omega - \hat{H}_0} \left(\omega - E + \hat{H}_I \right) \right\}^i | \Phi_0 \rangle =$$

$$\langle \Phi_0 | \left(\hat{H}_I + \hat{H}_I \frac{\hat{Q}}{E - \hat{H}_0} \hat{H}_I + \hat{H}_I \frac{\hat{Q}}{E - \hat{H}_0} \hat{H}_I \frac{\hat{Q}}{E - \hat{H}_0} \hat{H}_I + \dots \right) | \Phi_0 \rangle.$$

Electromagnetic multipole moments and transitions

$$\Delta E = \sum_{i=0}^{\infty} \langle \Phi_0 | \hat{H}_l \left\{ \frac{\hat{Q}}{\omega - \hat{H}_0} \left(\omega - E + \hat{H}_l \right) \right\}^i | \Phi_0 \rangle = \\
\langle \Phi_0 | \left(\hat{H}_l + \hat{H}_l \frac{\hat{Q}}{E - \hat{H}_0} \hat{H}_l + \hat{H}_l \frac{\hat{Q}}{E - \hat{H}_0} \hat{H}_l \frac{\hat{Q}}{E - \hat{H}_0} \hat{H}_l + \dots \right) | \Phi_0 \rangle.$$
This expression depends however on the exact energy *E* and is again not very convenient from a practical point of view. It can obviously be solved iteratively, by starting with a guess for *E* and then solve till some kind of self-consistency criterion has been reached. Actually, the above expression is nothing but a rewrite again of the full Schrödinger equation.

Defining $e = E - \hat{H}_0$ and recalling that \hat{H}_0 commutes with \hat{Q} by construction and that \hat{Q} is an idempotent operator $\hat{Q}^2 = \hat{Q}$. Using this equation in the above expansion for ΔE we can write the denominator

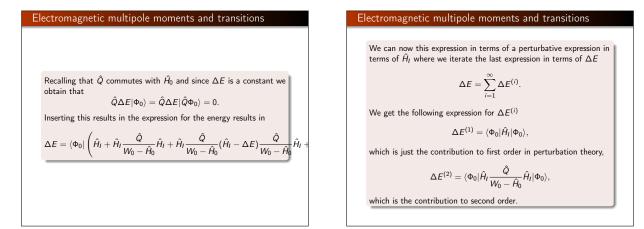
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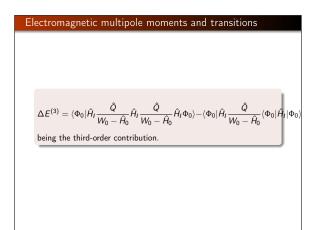
Electromagnetic multipole moments and transitions
Inserted in the expression for
$$\Delta E$$
 leads to

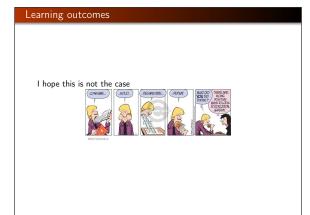
$$\Delta E = \langle \Phi_0 | \hat{H}_l + \hat{H}_l \hat{Q} \frac{1}{E - \hat{H}_0 - \hat{Q} \hat{H}_l \hat{Q}} \hat{Q} \hat{H}_l | \Phi_0 \rangle.$$
In RS perturbation theory we set $\omega = W_0$ and obtain the following expression for the energy difference

$$\Delta E = \sum_{i=0}^{\infty} \langle \Phi_0 | \hat{H}_l \left\{ \frac{\hat{Q}}{W_0 - \hat{H}_0} \left(\hat{H}_l - \Delta E \right) \right\}^i | \Phi_0 \rangle =$$

$$\langle \Phi_0 | \left(\hat{H}_l + \hat{H}_l \frac{\hat{Q}}{W_0 - \hat{H}_0} (\hat{H}_l - \Delta E) + \hat{H}_l \frac{\hat{Q}}{W_0 - \hat{H}_0} (\hat{H}_l - \Delta E) \frac{\hat{Q}}{W_0 - \hat{H}_0} (\hat{H}_l - \Delta E) \right]$$







Topics we have covered this year

- Single-particle properties and mean-field and relation to data
- How to set up basis states in second quantization and find expectation values
- Angular momentum properties and the Wigner-Eckart theorem
- Short survey of properties of nuclear forces
- The nuclear shell model
- And how to relate a shell-model calculation to decays and properties of decays.

Final presentation

- Introduction with motivation
- Explain an eventual experimental set up
- Give a short overview of the theory employed and how it relates to the analysis of eventual data
- Present and discuss your results
- Summary, conclusions and perspectives
- Anything else you think is important. Useful to have backup slides

In total your talk should have a duration of 20-25 minutes, but longer is also ok.

